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EVALUATION OF MINE THREAT

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NORTH ATLANTIC TREATY ORGANIZATION

Evaluation of mine threat

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The content of this document pertains to work performed under Project 053-3 of the SACLANTCEN Programme of Work. The document has been approved for release by The Director, SACLANTCEN.

David L. Bradley
Director

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Evaluation of mine threat

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Executive Summary: This document has been prepared by the Operational Research Group at SACLANTCEN for the Mine Countermeasures (MCM) Planning and Evaluation (P&E) Ad Hoc Working Group (AHWG). It reports the work carried out in support of the CINCEASTLANT proposal entitled 'Methods of Assessing the Mine Risk to Merchant Shipping and Naval Operations', dated 3 April 1995 under Project 053-3 in the Command Support thrust area of SACLANTCEN's scientific programme of work. The overall aim of the SACLANTCEN study task was to define algorithms to provide accurate, quantitative assessments of the risk to shipping from mines.

The specific objective of the work described in this report was defined by the AHWG as the development of methods of incorporating ship count distributions into the calculation of threat to target vessels and expected casualties prior to and after the execution of MCM operations. As such, these methods are proposed for incorporation within the NATO MCM EXclusive Planning Evaluation Risk Tool (EXPERT) software system. The task was to consider both:

- The mine threat to targets transiting along and constrained to channels.
- The mine threat to targets patrolling within an area.

This document presents the approach adopted by SACLANTCEN and provides a detailed mathematical description of the methods developed to evaluate the mine threat. A comparison is made between the results predicted by the developed algorithms and those produced by a Monte Carlo simulation. For the limited situations examined, there is good agreement between the analytical and simulation results. Therefore, it is concluded that the proposed algorithms accurately predict the mine threat.

Annex A to this document provides an algorithmic description for the implementation of the methods within MCM EXPERT, a definition of the input and output parameters, and a data dictionary. Annex B contains the source code for a Microsoft Visual Basic implementation of the algorithms.

It is recommended that the algorithms are incorporated into the MCM EXPERT system. Additionally, the algorithms given in this document may form the basis for improved definitions of minefield measures of effectiveness (MOEs).

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Evaluation of mine threat

J.C.J Redmayne

Abstract: This report describes methods for incorporating ship count distributions into the calculation of both the mine threat to target vessels and the number of expected casualties, prior to and after the execution of mine countermeasures (MCM) operations. The theory is developed for the scenarios of a target transiting along and constrained to a channel, and a target patrolling within an area. Detailed mathematical descriptions are provided of the methods developed to evaluate the mine threat. A comparison is made between the results predicted by the proposed algorithms and those produced by a Monte Carlo simulation. Algorithmic descriptions are given suitable for computer implementation.

Keywords: mine countermeasures ∘ MCM ∘ mine threat ∘ ship count ∘ Simple Initial Threat ∘ SIT

Contents

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Introduction

This document has been prepared by the Operational Research Group (ORG) at SACLANTCEN for the Mine Countermeasures (MCM) Planning and Evaluation (P&E) Ad Hoc Working Group (AHWG). It reports the work carried out in support of the CINCEASTLANT proposal entitled 'Methods of Assessing the Mine Risk to Merchant Shipping and Naval Operations', dated 3 April 1995. This proposal is incorporated within the Command Support thrust area as Project 053-3 of the SACLANTCEN Scientific Programme of Work.

The overall aim of the SACLANTCEN study task is to define algorithms to provide accurate, quantitative assessments of the risk to shipping from mines based on a 'usable' data input set. Risk is the probability (generally expressed as a percentage) that the target vessel actuates a mine and the resultant shock from the mine detonation causes a defined level of damage to the target vessel e.g. the target suffers 'mission abort' damage. A usable data set is defined as one that is understandable to the military operator, uses standard parameters as defined in the relevant Allied Tactical Publications (References [1] and [2]) and for which the data inputs can be readily provided or calculated.

The specific objective of the work detailed in this document was defined by the AHWG (Reference [3]) as the development of methods of incorporating ship count distributions into the calculation of threat to target vessels and expected casualties prior to and after the execution of MCM operations As such, these methods are proposed to replace those contained in Reference [4] within the NATO MCM EXclusive Planning Evaluation Risk Tool (EXPERT) software system. The task was to consider both:

- The mine threat to targets transiting along and constrained to channels.
- The mine threat to targets patrolling within an area.

This report is divided into six sections and two annexes. Following this introduction, Section 2 develops the theory for the incorporation of ship counts into the calculation of mine threat to the first target vessel to transit along a mined channel. Section 3 then considers the problem of multiple target transits along a channel, where each target has an associated mine actuation width and the probability distributions of the target vessels across the channel are correlated. Section 4 addresses area operations and an algorithm is developed that incorporates ship counts and permits the mine threat to one or more target vessels patrolling an area to be determined. Section 5 contains the results produced by a Monte Carlo simulation model that was designed to validate the analytical algorithms presented in Sections 2 and 3. Section 6 presents the conclusions. Finally,

Annex A and Annex B at the back of this report are provided principally for the developers of the MCM EXPERT program, but may be useful for any computer implementation of the proposed algorithms. Annex A contains a high level description of the algorithms, the data dictionary and the input and output parameters. Annex B gives a listing of the Microsoft Visual Basic source code developed by SACLANTCEN during the verification of the algorithms (Section 5).

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Initial mine threat in channels

2.1 INTRODUCTION

This section develops the theory for incorporating ship count distributions into the calculation of the initial threat from mines to a target vessel constrained to a channel.

2.2 SIMPLE INITIAL THREAT

The simple initial threat (SIT) describes the threat posed to the first target vessel to perform a single transit through a mined channel (Reference [4]):

$$SIT = T_i = \left[1 - \left\{1 - \frac{W_d}{C}\right\}^i\right] \tag{1}$$

where in Eq. (1):

- W_d is the integral over the damage width of the mine actuation probability resulting from the mine-target interaction.
- C is the width of the channel.
- i is the number of mines within the channel.

The SIT assumes that:

a) Mine-target encounters in the direction of transit are independent.

b) All the i mines are located within the channel and are randomly (uniformly) distributed across the channel.

c) If the distance across the channel from the centre of the channel is defined as y ($y = 0$ being the channel centreline), the target vessel always remains within the region: $-\frac{C}{2} + \frac{W_d}{2} \le y \le \frac{C}{2} - \frac{W_d}{2}$. This assumption is generally valid as the standard NATO definition for C is $6\sigma_{ship} + W_d$, where σ_{ship} is the standard deviation of navigational error (SDNE) of the target vessel.

d) All the mines are poised i.e. all mines are armed and on ship count 1[1].

2.3 MODIFIED SIMPLE INITIAL THREAT

Reference [4] refines the SIT (Eq. (1)) to eliminate assumptions (b) and (c):

$$T_i = 1 - \left[1 - \int_{-\frac{C}{2}}^{+\frac{C}{2}} \left\{ \frac{m(y).(1 - PC(y)).\int_{y-\frac{W_d}{2}}^{y+\frac{W_d}{2}} s(u)du}{\int_{-\frac{C}{2}}^{+\frac{C}{2}} m(t).(1 - PC(t))dt} \right\} dy \right]^i \tag{2}$$

In Eq. (2):

m(y) is the *a priori* probability density function (pdf) of a mine being at across-channel position *y*.

PC(y) is the probability of removing a mine at across-channel position *y* i.e. *PC(y)* is the percentage clearance expressed as a probability.

s(u) is the pdf of a target vessel being at across-channel position *u*.

The integral:

$$g(y) = \int_{y-\frac{W_d}{2}}^{y+\frac{W_d}{2}} s(u)du \tag{3}$$

is the probability that a mine located at an across-channel position *y* encounters a target vessel within the damage width of the mine W_d. If the pdf of the target vessel being at across-channel position *y* (*s(u)*) is assumed to be a Gaussian distribution with zero mean (i.e. the target attempts to transit down the centreline of the channel) and standard deviation σ_{ship}, then *g(y)* is:

$$g(y) = \Phi\left(\frac{y + \frac{W_d}{2}}{\sigma_{ship}}\right) - \Phi\left(\frac{y - \frac{W_d}{2}}{\sigma_{ship}}\right)$$

[1] Within NATO it is generally assumed that ship counts decrement but this is not true in general; they may increment.

where $\Phi(Z)$ is the value of the cumulative standard normal distribution function at Z.

The function:

$$m'(y) = \frac{m(y).(1 - PC(y))}{\int_{-\frac{C}{2}}^{+\frac{C}{2}} m(t).(1 - PC(t))dt}$$

represents the *a posteriori* pdf of a mine being at across-channel position y after MCM effort has been carried out since the *a priori* mine pdf *m(y)* of a mine being at across-channel position y is modified by the clearance achieved across the channel by MCM operations. If the *a priori* mine pdf (*m(y)*) is assumed to be uniform (or random) over the width of the channel, such that $m(y) = \frac{1}{C}$, then:

$$m'(y) = \frac{(1 - PC(y))}{\int_{-\frac{C}{2}}^{+\frac{C}{2}} (1 - PC(t))dt} \tag{4}$$

Eq. (2) can be simplified as:

$$T_i = 1 - \left[1 - \int_{-\frac{C}{2}}^{+\frac{C}{2}} m'(y).g(y)dy \right]^i \tag{5}$$

2.4 SHIP COUNTS

If mines on ship counts of greater than 1 are encountered, then Eq. (5) requires modification as only the mines that are on ship count 1 represent a threat to the first target vessel (the initial threat). Define *L(k,y)* as the probability that a mine at across-channel position y is on ship count k; k is an integer from 0 to K_{max}, where K_{max} is the maximum ship count expected. *L(0,y)* is the probability that a mine is on ship count 0, i.e. it has been actuated, and hence $L(0, y) = PC(y)$.[2]

The probability that a mine at position y is on ship count 1 is *L(1,y)*. Incorporating this parameter into Eq. (5), the initial threat to the first target vessel is:

[2] Note that the convention for L used in this report differs from that given in Reference [5] as the order of y and k has been altered to be consistent with the conventions used in Reference [6].

$$T_i = 1 - \left[1 - \int_{-\frac{C}{2}}^{+\frac{C}{2}} m'(y).\, L(1,y).\, g(y)dy \right]^i \tag{6}$$

where i is the number of mines remaining in the channel.

2.5 DETERMINATION OF L(k,y)

The non-uniform coverage evaluation (NUCEVAL - Reference [6]) module developed by the US and implemented within the MCM EXPERT software calculates the percentage clearance (i.e. the probability of mine removal by MCM) as a fraction *PC(S,IY,I)* for mine type *I*, initially on ship count *S* and at the discrete across-channel position *IY*. Considering a single mine type only thus dropping the mine type *I* subscript, and letting the discrete across-channel position *IY* be a continuous position *y*, let the fractional clearance for a mine initially on ship count *S* and at position *y* be *PC(S,y)*.

The percentage clearance is defined as the probability of receiving **at least** *S* actuations. Assume that a mine is initially on ship count *S*. The probability that it is now on ship count *k* is dependent on it receiving **exactly** *S-k* actuations. The probability of a mine receiving **exactly** *S-k* actuations is:

Probability of receiving exactly S-k actuations
= *Probability of receiving at least S-k actuations - Probability of receiving at least S-k+1 actuations*

$$= PC(S-k,y) - PC(S-k+1,y) \tag{7}$$

For $k = S$, *PC(0,y)* represents the clearance probability for mines originally on ship count 0 i.e. actuated , and therefore, by definition for all values of y ($\forall\, y$):

$$(\forall\, y) \quad (PC(0,y) = 1) \tag{8}$$

To find the probability of a mine being present in the channel and on ship count k (*L(k,y)*), it is now necessary to normalise:

$$L(k,y) = \frac{PC(S-k,y) - PC(S-k+1,y)}{\sum_{j=1}^{S} PC(S-j,y) - PC(S-j+1,y)} \tag{9}$$

but $\sum_{j=1}^{S} PC(S-j,y) - PC(S-j+1,y) = 1 - PC(S,y)$ and therefore:

$$L(k,y) = \frac{PC(S-k,y) - PC(S-k+1,y)}{1 - PC(S,y)} \tag{10}$$

Finally:

$$L(1,y) = \frac{PC(S-1,y) - PC(S,y)}{1 - PC(S,y)} \tag{11}$$

Note that after the normalisation process $L(0,y) = 0$.

2.6 INITIAL THREAT INCLUDING SHIP COUNTS

For mines initially on ship count S, the *a posteriori* pdf ($m'(y)$) of a mine being at across-channel position y is determined using the fractional clearance for mines on ship count S, $PC(S,y)$ in Eq. (4):

$$m'(y) = \frac{(1 - PC(S,y))}{\int_{-\frac{C}{2}}^{+\frac{C}{2}} (1 - PC(S,t))dt} \tag{12}$$

Eq. (6) may be modified to give the final expression for the threat posed by i mines all initially set (prior to MCM effort being expended) on ship count S ($T_i(S)$):

$$T_i(S) = 1 - \left[1 - \int_{-\frac{C}{2}}^{+\frac{C}{2}} m'(y).\, L(1,y).\, g(y)dy \right]^i \tag{13}$$

Substituting for $m'(y)$ (Eq. (12)), $L(1,y)$ (Eq. (11)) and $g(y)$ (Eq. (3)):

$$T_i(S) = 1 - \left[1 - \int_{-\frac{C}{2}}^{+\frac{C}{2}} \left\{ \frac{(1 - PC(S,y)).\dfrac{\{PC(S-1,y) - PC(S,y)\}}{\{1 - PC(S,y)\}}.\displaystyle\int_{y-\frac{W_d}{2}}^{y+\frac{W_d}{2}} s(u)du}{\displaystyle\int_{-\frac{C}{2}}^{+\frac{C}{2}} (1 - PC(S,t))dt} \right\} dy \right]^i \tag{14}$$

Simplifying:

$$T_i(S) = 1 - \left[1 - \int_{-\frac{C}{2}}^{+\frac{C}{2}} \left\{ \frac{\{PC(S-1,y) - PC(S,y)\}.\int_{y-\frac{W_d}{2}}^{y+\frac{W_d}{2}} s(u)du}{\int_{-\frac{C}{2}}^{+\frac{C}{2}} (1 - PC(S,t))dt} \right\} dy \right]^i \tag{15}$$

2.7 PROBABILITY OF MINES REMAINING

As proposed in Reference [4], the probability of *i* mines remaining in a channel may be determined using Bayesian techniques. These techniques use the average percentage clearance achieved across the channel and the number of mines countered. For mines initially on ship count *S*, the average percentage clearance achieved across the channel $\overline{PC(S)}$ is:

$$\overline{PC(S)} = \frac{1}{C} \int_{-\frac{C}{2}}^{+\frac{C}{2}} PC(S,y)\,dy \tag{16}$$

Using $\overline{PC(S)}$ in the Bayesian calculations produces an *a posteriori* probability for *i* mines remaining if all mines were initially on ship count *S* ($P_i(S)$).

2.8 INITIAL RISK TO SHIPPING

The expected risk to the first target vessel from mines initially on ship count *S* (*R(S)*) is therefore:

$$R(S) = \sum_i P_i(S).T_i(S) \tag{17}$$

2.9 INITIAL SHIP COUNT DISTRIBUTION

The expected initial ship count distribution (*SCDIST(S)*) prior to MCM must be specified by the user, where $1 \le S \le K_{\max}$. Suggested distributions for user selection within the MCM EXPERT program are:

- $(1 \leq S \leq K_{\max} - 1)$ $(SCDIST(S) = 0)$; $(S = K_{\max})$ $(SCDIST(S) = 1)$ i.e. all mines on the maximum ship count K_{max},
- $(1 \leq S \leq K_{\max})$ $(SCDIST(S) = \frac{1}{K_{\max}})$ i.e. uniform distribution of ship counts.

2.10 OVERALL RISK

The overall risk (*R*) to the first target vessel is found by weighting the risk from mines on ship count *S* (*R(S)*) by the initial ship count distribution *SCDIST(S)*:

$$R = \sum_{S=1}^{K_{\max}} SCDIST(S) \,.\, R(S) \qquad (18)$$

3

Mine threat to multiple target vessels

3.1 INTRODUCTION

This section builds upon the concepts developed in Section 2 with the objectives of defining the threat to multiple target vessels transiting along a mined channel and hence finding the expected number of target casualties. It is assumed that associated with each target vessel is a mine actuation width which is not necessarily equal to the mine damage width.

3.2 DEFINITIONS

Let:

T	be the total number of targets to transit through the mined channel.
$T_i^{(\tau)}(S)$	be the mine threat to τ^{th} target vessel posed by i mines all initially on ship count S.
$CAS^{(T)}(S)$	be the expected number of casualties after T transits along the channel caused by mines all initially on ship count S.
W_a	be the aggregate actuation width of the target vessel i.e. the integral of the mine actuation probability resulting from the mine-target interaction.

3.3 THREAT TO TRANSITORS

The initial threat to the first transitor is given by Eq. (15). The threat to subsequent target vessel transits is dependent on the number of mines actuated (i.e. exploded or ship count decremented) by each target transit; if the actuation width W_a is greater than the 'mission abort' damage width W_d, then it is possible for mines to be actuated without causing 'mission abort' damage the target vessel. Moreover, two situations need to be taken into account:

a) The 'mission abort' criterion used to determine the damage width does not include immobilisation of the target vessel, and the vessel is assumed to continue and complete its transit through the mined channel, possibly actuating additional mines.

b) The 'mission abort' criterion is such that a damaged vessel cannot continue its transit and therefore cannot actuate additional mines[3].

These two situations are examined in the following sections.

3.3.1 Target is not immobilised

In this situation, each target vessel is assumed to complete its transit through the mined channel, no matter how many mines are actuated within the damage width of the vessel. This situation is probably not realistic as there is likely to be some cumulative effect from multiple shocks from mine explosions. However, the number of mines 'swept' and removed by each transitor is maximised. Each transitor may be considered to be a minesweeper with an aggregate actuation width W_a.

For a mine at across-channel position y, the probability $h(y)$ of the mine being actuated by the target vessel is:

$$h(y) = \int_{y-\frac{W_a}{2}}^{y+\frac{W_a}{2}} s(u)\,du \tag{19}$$

If the pdf of the target vessel being at across-channel position u ($s(u)$) is assumed to be a Gaussian distribution with zero mean (i.e. the target attempts to transit down the centreline of the channel) and standard deviation σ_{ship}, then $h(y)$ is:

$$h(y) = \Phi\left(\frac{y+\frac{W_a}{2}}{\sigma_{ship}}\right) - \Phi\left(\frac{y-\frac{W_a}{2}}{\sigma_{ship}}\right) \tag{20}$$

where $\Phi(Z)$ is the value of the cumulative standard normal distribution function at Z.

This actuation probability $h(y)$ operates on the ship count probability vector $L(k,y)$; after the first target transit, the ship count is updated as follows:

$$(\forall\ y) \quad (0 \le k \le S) \quad L^{(1)}(k,y) = L^{(0)}(k+1,y).h(y) + L^{(0)}(k,y).(1-h(y)) \tag{21}$$

The following initial states apply to the ship count probability vector $L^{(0)}(k,y)$ for mines initially on ship count S prior to the first target vessel transit:

[3] Reality probably lies somewhere between these two extremes.

$$(\forall\, y) \quad L^{(0)}(S+1,y)=0$$

$$(\forall\, y) \quad L^{(0)}(0,y)=0$$

$$(\forall\, y) \quad (1 \le k \le S)\; L^{(0)}(k,y) \text{ is calculated using Eq. (10).}$$

Let the ship count probability vector after the τ^{th} transit be $L^{\tau}(k,y)$; $L^{\tau}(k,y)$ is determined after each transit using Eq. (21):

$$(\forall\, y) \quad (0 \le k \le S) \quad L^{\tau}(k,y) = L^{\tau-1}(k+1,y).h(y) + L^{\tau-1}(k,y).(1-h(y)) \qquad (22)$$

So the threat to the $\tau+1$ th transit is given by substituting $L^{\tau}(1,y)$ in place of $L(1,y)$ in Eq. (13):

$$T_i^{(\tau+1)}(S) = 1 - \left[1 - \int_{-\frac{C}{2}}^{+\frac{C}{2}} m'(y).\, L^{\tau}(1,y).\, g(y)dy \right]^i \qquad (23)$$

3.3.2 Target is immobilised

For this situation, it is assumed that a mine actuation within the damage width W_d of the target vessel results in immobilisation of the vessel such that no further mines may be actuated. However, mines may be actuated outside of the damage width until a mine strike occurs, if at all, within the damage width. For this situation, the number of mines 'swept' and removed by each transitor will be minimised. Again, the vessel may be considered to be a minesweeper until either the transit is completed or immobilisation, whichever occurs first.

Consider i mines, all initially on ship count S, to be ordered according to their distance along the channel in the direction of the transit. This ordering does not affect the assumptions of the mines being randomly distributed across the channel (Section 2.3) and the mine-target encounters being independent. Numbering the mines according to their order along the channel (*1, 2, 3, ... i-1, i*), the probability that the first vessel has the opportunity to encounter the l^{th} mine is $1 - T_{l-1}^{(1)}(S)$ [4]. The probability that the l^{th} mine is actuated is $h(y)$. Therefore, the average mine actuation probability for any one of i mines located at any point along the channel and at across-channel position y is:

[4] Note that $1 - T_{l-1}^{(1)}(S) = 1 - T_{l-1}(S)$, the threat to the first transitor (Eq. (13)).

$$h^{1}(y)=\frac{1}{i}\sum_{l=1}^{i}[1-T_{l-1}^{(1)}(S)].h(y)=\frac{1}{i}\sum_{l=1}^{i}[1-T_{1}^{(1)}(S)]^{l-1}.h(y)$$

Where $T_1^{(1)}(S)$ is the threat to the first transit from a single mine. Summing the geometric progression to i terms:

$$h^{1}(y)=\frac{1}{i}.\left\{\frac{1-(1-T_{1}^{(1)}(S))^{i}}{T_{1}^{(1)}(S)}\right\}.h(y) \qquad (24)$$

It is apparent that the mine actuation probability is now a function of the threat to the transitor; thus, for the τ^{th} transit, the average mine actuation probability for a mine at across-channel position y is:

$$h^{\tau}(y)=\frac{1}{i}.\left\{\frac{1-(1-T_{1}^{(\tau)}(S))^{i}}{T_{1}^{(\tau)}(S)}\right\}.h(y) \qquad (25)$$

Thus, $h^{\tau}(y)$ is recalculated for each transit and is then used to update the ship count probability vector $L^{\tau}(k,y)$ by replacing $h(y)$ with $h^{\tau}(y)$ in Eq. (22). The threat to the $\tau+1$ th transit is then found as in Section 3.3.1 using Eq. (23). Note that $h^{0}(y)$ equals $h(y)$.

3.4 RISK TO SHIPPING

The expected risk to the τ^{th} target vessel from mines initially on ship count S ($R^{(\tau)}(S)$) is therefore:

$$R^{(\tau)}(S)=\sum_{i}P_{i}(S).T_{i}^{(\tau)}(S) \qquad (26)$$

3.5 EXPECTED CASUALTIES

The expected number of casualties after T transits caused by mines initially on ship count S is:

$$CAS^{(\mathrm{T})}(S)=\sum_{\tau=1}^{\mathrm{T}}R^{(\tau)}(S) \qquad (27)$$

3.6 OVERALL RISK

The overall risk ($R^{(\tau)}$) to the τ^{th} target vessel is found by weighting the risk from mines on ship count S ($R^{(\tau)}(S)$) by the initial ship count distribution $SCDIST(S)$:

$$R^{(\tau)} = \sum_{S=1}^{K_{\max}} SCDIST(S) \,.\, R^{(\tau)}(S) \tag{28}$$

3.7 OVERALL EXPECTED CASUALTIES

The overall expected casualties after T transits is found by weighting the expected casualties from mines on ship count S by the initial ship count distribution $SCDIST(S)$:

$$CAS^{(\mathrm{T})} = \sum_{S=1}^{K_{\max}} SCDIST(S) \,.\, CAS^{(\mathrm{T})}(S) \tag{29}$$

4

Mine threat to patrolling targets

4.1 INTRODUCTION

This section develops a method for the calculation of mine threat to a target vessel carrying out a 'patrol' within an allocated area. The target is assumed to move randomly within the defined area.

4.2 DEFINITIONS

Let:

- v be the speed of the target vessel.
- t be the time spent patrolling within the area.
- F be the total area of the patrol region.

4.3 PROBABILITY OF TARGET ACTUATING A MINE

The target vessel will actuate a mine if a mine encounter occurs within the actuation width W_a. Consider a single mine located somewhere within the region of area F. The average number of times (μ) that the target will encounter this single mine in a time t is:

$$\mu = \frac{W_a . v . t}{F} \tag{30}$$

Therefore, μ represents the mean number of mine actuations of the single mine occurring in time t. Assuming that the target's movement can be modelled as a random process, then the number of actuations of the mine follows Poisson probability distribution with parameter μ. The probability of r mine actuations occurring is hence described by:

$$p(r) = \frac{e^{-\mu} \mu^r}{r!} \tag{31}$$

If the mine is on ship count k, then the target needs to actuate the mine **at least** k times to achieve mine detonation. The probability of at least k actuations is:

$$p(r \geq k) = \sum_{r=k}^{\infty} \frac{e^{-\mu} \mu^r}{r!}$$

Or, in terms of the cumulative Poisson distribution:

$$p(r \geq k) = 1 - \sum_{r=0}^{k-1} \frac{e^{-\mu} \mu^r}{r!} \tag{32}$$

To damage the target vessel, the final mine actuation must occur within the damage width. The probability of this happening is the ratio of W_d to W_a. Therefore, the threat posed by a single mine on ship count k is:

$$T_1(k) = \frac{W_d}{W_a} \left\{ 1 - \sum_{r=0}^{k-1} \frac{e^{-\mu} \mu^r}{r!} \right\} \tag{33}$$

4.4 SHIP COUNT PROBABILITIES

If the mine is originally on ship count S, then the probability that a mine is on ship count k after MCM operations have been carried out is described by $L(k)$. Note that the y subscript is not required because the distribution of the target vessel is assumed to be random throughout the area F and is not related to across-channel position.

Let the average percentage clearance across the whole area F for a mine initially on ship count S be $\overline{PC(S)}$. Then $L(k)$ is found using a modified form of Eq. (10):

$$L(k) = \frac{\overline{PC(S-k)} - \overline{PC(S-k+1)}}{1 - \overline{PC(S)}} \tag{34}$$

Similar to Eq. (8), note that by definition $\overline{PC(0)} = 1$.

4.5 MINE THREAT TO PATROLLER

Therefore, the threat now posed by a single mine initially on ship count S is found by combining Eq. (33) and Eq. (34):

$$T_1(S) = \sum_{k=1}^{S} L(k).\frac{W_d}{W_a}\left\{1 - \sum_{r=0}^{k-1}\frac{e^{-\mu}\mu^r}{r!}\right\}$$
$$= \sum_{k=1}^{S}\left\{\frac{\overline{PC(S-k)} - \overline{PC(S-k+1)}}{1-\overline{PC(S)}}\right\}.\frac{W_d}{W_a}\left\{1 - \sum_{r=0}^{k-1}\frac{e^{-\mu}\mu^r}{r!}\right\} \tag{35}$$

Assuming that there are i mines in the region F, the threat from i mines is:

$$T_i(S) = 1 - [1 - T_1(S)]^i \tag{36}$$

4.6 INITIAL RISK TO SHIPPING

The expected risk to the patrolling target vessel from mines initially on ship count S ($R(S)$) is therefore:

$$R(S) = \sum_i P_i(S).T_i(S) \tag{37}$$

4.7 OVERALL RISK

The *a priori* ship count distribution (*SCDIST(S)*) is entered by the user from ship counts from 1 to K_{max}. The overall risk (R) to the first target vessel is found by weighting the risk from mines on ship count S ($R(S)$) by the initial ship count distribution *SCDIST(S)*:

$$R = \sum_{S=1}^{K_{max}} SCDIST(S).R(S) \tag{38}$$

4.8 MORE THAN ONE PATROLLER

If there is more than one target vessel patrolling the region F, and assuming that the vessels operate independently, the risk that at least one vessel will suffer damage in time t can be evaluated using the techniques presented for a single ship in Sections 4.3 to 4.7 above, but the value of μ in Eq. (30) is modified to:

$$\mu = \frac{N_{ships}.W_a.v.t}{F} \tag{39}$$

where, N_{ships} is the number of independent vessels operating in the region.

5

Algorithm verification

5.1 INTRODUCTION

This section contains the results of a short study carried out with the objective of verifying the algorithms proposed in this document. This study was limited to checking the threat to a transitor posed by *i* mines all initially on ship count *S* ($T_i^{(\tau)}(S)$ as defined by Eq. (23)). The verification process was performed by writing a Monte Carlo simulation model and then comparing the threats to each transitor from the simulation with those predicted by the analytical algorithms.

Three different minesweeping operations were examined:

- A time limited non-uniform coverage operation that was successfully completed.
- A time limited non-uniform coverage operation that was only partially completed.
- A completed uniform coverage operation.

For each operation, the NUCEVAL algorithms were used to determine the clearance across the channel for mines initially on ship count *S* (*PC(S,y)*).

5.2 MCM PARAMETERS

The constant parameters assumed during the verification process are detailed in Table 1.

Table 1 - Parameters assumed for algorithm verification

Parameter	**Value**
Number of calculation points across channel (*NY*)	201
Channel width (*C*)	1000 m
Calculated resolution (*DY*)	5.0 m
Characteristic actuation probability (*B*)	0.9
Characteristic actuation width (*A*)	200 m
MCMV standard deviation of navigational error (SDNE)	25 m
Maximum ship count (K_{max})	5
Ship count distribution	UNIFORM

Table 1 (continued)

Parameter	**Value**
Number of mines (n)	10
Mine damage width (W_d)	100 m
Target mine actuation width (W_a)	150 m
Target shipping across channel distribution type	NORMAL
Target shipping SDNE (σ_{ship})	100 m
Immobilise on mine strike?	YES
Total number of transitors (T)	10
Number of iterations of simulation for each ship count	5000

For the completed non-uniform coverage minesweeping operation, a total of 10 tracks were planned; the track positions are detailed in Table 2 (0 m indicates the channel centreline, negative values are to port and positive to starboard). For the incomplete non-uniform coverage task, the same track locations were assumed but the number of runs per track along the channel centreline was reduced from 8 to 4. Similarly, Table 3 details the track data for the uniform coverage task which planned for an average (across the channel width and with respect to all the ship counts from 1 to 5) percentage clearance of at least 80%, assuming a uniform *a priori* ship count distribution.

Table 2 - Non-uniform coverage tasks

Track Position	**Runs per Track (Completed)**	**Runs per Track (Incomplete)**
-100 m	1	1
0 m	8	4
100 m	1	1

Table 3 - Uniform coverage task

Track Position	**Runs per Track**
-458.3 m	2
-375.0 m	2
-291.7 m	2
-208.3 m	2
-125.0 m	2
-41.7 m	2
41.7 m	2
125.0 m	2
208.3 m	2
291.7 m	2
375.0 m	2
458.3 m	2

5.3 ALGORITHM VERIFICATION RESULTS

The results for the algorithm verification runs are illustrated in Figure 1, Figure 2 and Figure 3, which show charts of the threat to the target vessel versus the transit number.

A total of 5000 iterations of the Monte Carlo simulation were used for each of the individual tasks and ship counts. The error bars on the simulation series represent the 95% confidence level in the simulation results.

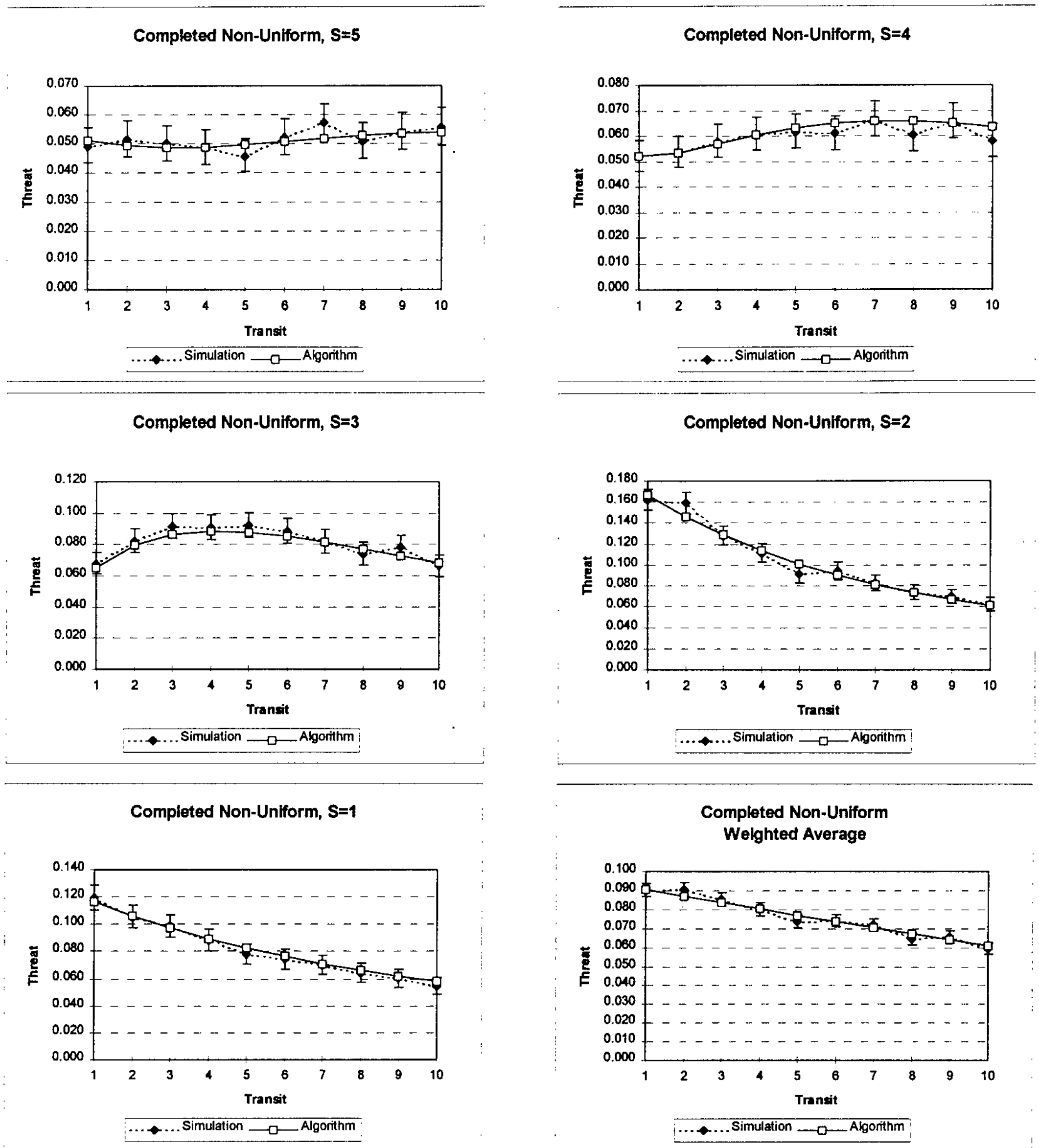

Figure 1 - Algorithm verification results for completed non-uniform coverage task

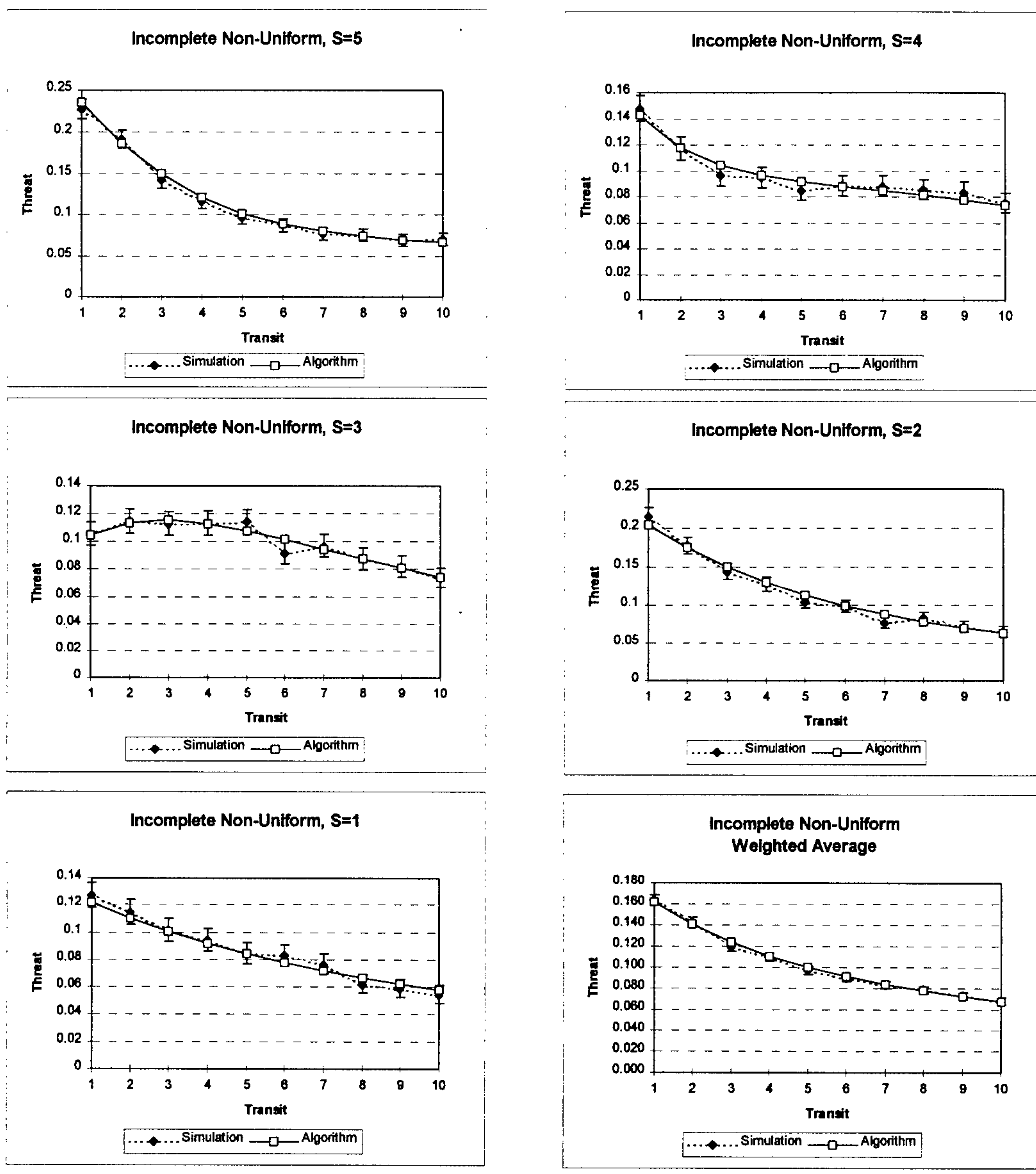

Figure 2 - Algorithm verification results for incomplete non-uniform coverage task

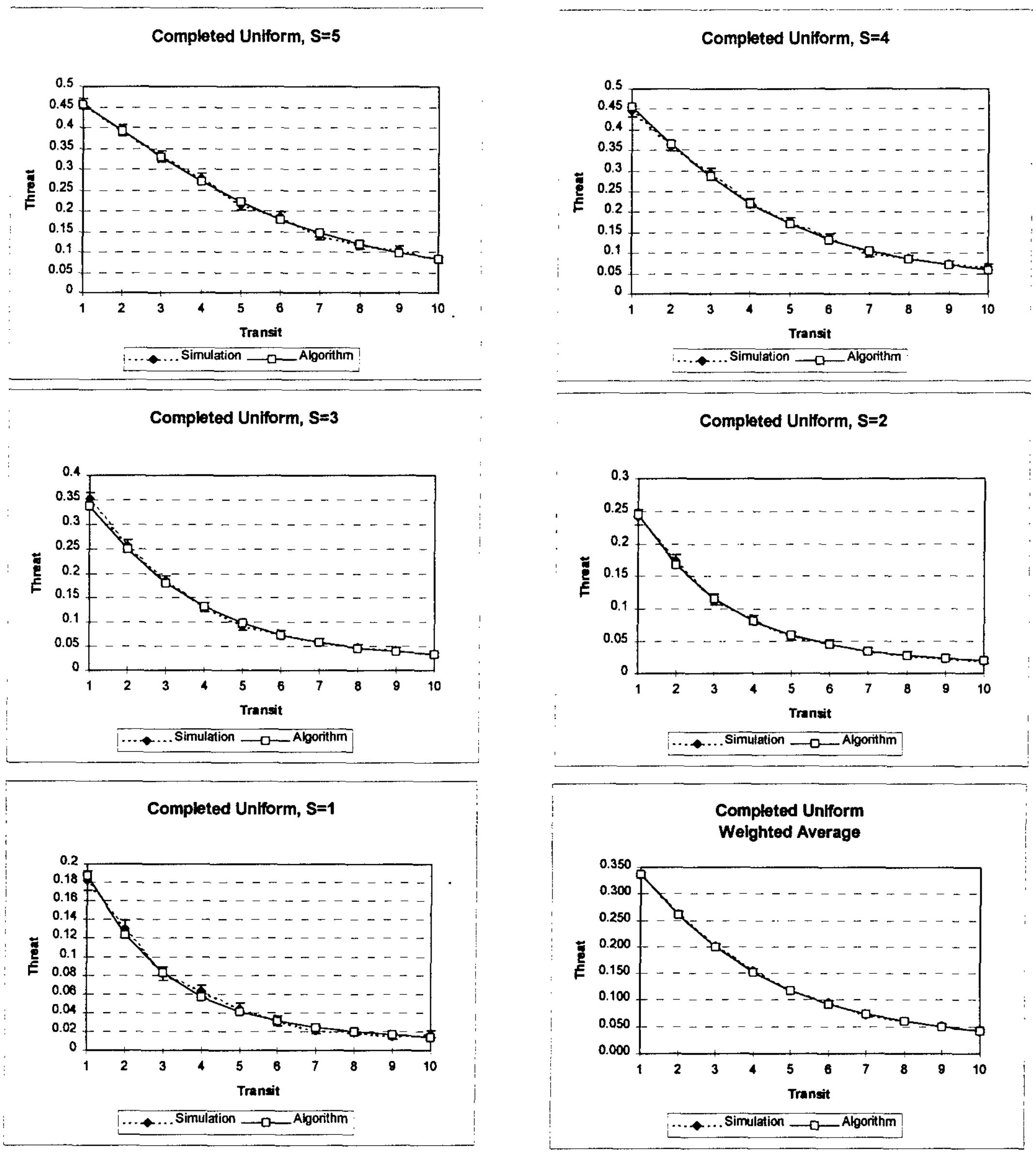

Figure 3 - Algorithm verification results for completed uniform coverage task

The weighted average results were obtained by assuming a uniform *a priori* distribution of ship counts. It can be seen from Figure 1 to Figure 3 that the algorithm results closely match the simulation results and are within the 95% confidence levels for all data points[5].

It is concluded that the algorithms developed accurately calculate the mine threat to target vessel transits along a channel.

5.4 DETAILED RESULTS FOR WEIGHTED AVERAGES

Detailed results for the weighted average data plotted in Figure 1, Figure 2 and Figure 3 are given in Table 4, Table 5 and Table 6, respectively, as test data for verification of the algorithms as implemented within MCM EXPERT. A uniform *a priori* distribution of ship counts was assumed for the weightings.

Table 4 - Weighted average threat for completed non-uniform coverage task

Transit Number	Algorithm Threat	Simulation Threat	Upper Confidence Limit (95%)	Lower Confidence Limit (95%)
Transit 1	0.090	0.090	0.094	0.087
Transit 2	0.087	0.090	0.094	0.087
Transit 3	0.084	0.085	0.089	0.082
Transit 4	0.080	0.080	0.083	0.077
Transit 5	0.077	0.074	0.077	0.071
Transit 6	0.074	0.074	0.077	0.071
Transit 7	0.070	0.072	0.075	0.069
Transit 8	0.067	0.064	0.068	0.062
Transit 9	0.064	0.065	0.069	0.062
Transit 10	0.061	0.059	0.062	0.056
Expected Casualties	0.754	0.754	-	-

[5] Note that the calculated threat is higher for the uniform coverage task because the **threat** calculation (as opposed to the **risk** calculation) takes no account of the probabilities of mines remaining within the channel. Hence, after uniform coverage, mines are equally likely at all points across the channel, whereas if the MCM effort is concentrated on the centreline (as in the non-uniform coverage situation) the algorithm assigns higher probabilities to the mines being at the channel edges. Mines situated at the edge of a channel pose a reduced threat to a target vessel attempting to transit down the channel centreline.

Table 5 - Weighted average threat for incomplete non-uniform coverage task

Transit Number	Algorithm Threat	Simulation Threat	Upper Confidence Limit (95%)	Lower Confidence Limit (95%)
Transit 1	0.162	0.164	0.166	0.160
Transit 2	0.141	0.143	0.145	0.139
Transit 3	0.124	0.119	0.128	0.115
Transit 4	0.110	0.109	0.114	0.105
Transit 5	0.100	0.097	0.104	0.093
Transit 6	0.091	0.090	0.095	0.086
Transit 7	0.084	0.083	0.087	0.080
Transit 8	0.078	0.079	0.081	0.075
Transit 9	0.072	0.073	0.076	0.070
Transit 10	0.067	0.068	0.071	0.065
Expected Casualties	1.029	1.025	-	-

Table 6 - Weighted average threat for completed uniform coverage task

Transit Number	Algorithm Threat	Simulation Threat	Upper Confidence Limit (95%)	Lower Confidence Limit (95%)
Transit 1	0.337	0.335	0.341	0.330
Transit 2	0.260	0.262	0.268	0.257
Transit 3	0.199	0.201	0.206	0.196
Transit 4	0.153	0.155	0.160	0.151
Transit 5	0.118	0.117	0.121	0.113
Transit 6	0.092	0.096	0.100	0.092
Transit 7	0.074	0.071	0.074	0.068
Transit 8	0.060	0.060	0.063	0.057
Transit 9	0.049	0.052	0.055	0.049
Transit 10	0.042	0.043	0.046	0.041
Expected Casualties	1.383	1.393	-	-

6
Conclusions

Methods have been developed to incorporate ship count distributions into the calculation of mine threat. The effect of MCM operations on the ship count states is considered in the proposed models. The methods allow the threat to be calculated for multiple target transits through a mined channel, with each target having an associated mine actuation width which may be different from the target's mine damage width. The navigational error of the targets and the correlation between the across-channel positions of separate target transits have also been considered.

A method has also been developed for the calculation of the mine threat to a target vessel carrying out a patrol within a defined area. The target's motion is assumed to be random such that the probability distribution of actuations of mines randomly located within the area follows a Poisson distribution.

The threat values predicted by the proposed algorithms have been compared with the results produced by a simple Monte Carlo simulation and good agreement has been found for the limited number of situations studied.

Although this work has been undertaken on behalf of the NATO MCM Planning and Evaluation AHWG for the purposes of the evaluation of risk to target shipping from mines prior to and subsequent to MCM operations, it is considered that the proposed algorithms have application in the definition of minefield measures of effectiveness (MOEs) and that they improve on the SIT formula, widely used to quantify minefield effectiveness at the current time.

References

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[4] Gerloch, J. Method for the evaluation of risk to shipping as utilised in MCM EXPERT Version 1.0, MWDC Working Note 69/95, NATO RESTRICTED. Portsmouth, UK, Maritime Warfare Development Centre, 19 September 1995.

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Annex A

Algorithmic description

A.1 INTRODUCTION

This section contains an algorithmic description for the methods presented in Sections 2 to 4, for implementation in the MCM EXPERT system.

A.2 DESCRIPTION FOR CHANNELS

The overall RISKEVAL algorithm for channel operations may be summarised as:

begin

collect data
initialise variables
evaluate the integral $g(y)$ using Eq. (3)

if number of transitors $T > 1$ then
    evaluate the integral $h(y)$ using Eq. (19)
end if

for each ship count S from 1 to K_{max}
    evaluate the integral $m'(y)$ using Eq. (12)
    initialise the ship count probability vector $L^{(0)}(k,y)$ using Eq. (10)

    for each mine remaining i from 0 to n
        calculate the *a posteriori* probability for i mines remaining if all mines were initially on ship count S ($P_i(S)$)

        for each transitor τ from 0 to $(T-1)$
            calculate the threat from one mine $T_1^{\tau+1}(S)$ using Eq. (23)
            calculate the threat from i mines $T_i^{\tau+1}(S)$ using Eq. (23)
            update the ship count probability vector $L^{(\tau)}(k,y)$ using Eq. (22)
            update the risk to transitor τ $R^{(\tau)}(S)$ from mines initially on ship count S using Eq. (26)

next transitor τ

next mine i

for each transitor τ from 1 to (T)

update the expected casualties $CAS^{(T)}(S)$ using Eq. (27)

next transitor τ

next ship count S

for each ship count S from 1 to K_{max}

calculate overall risk to transitor τ $R^{(\tau)}$ using Eq. (28)

calculate overall expected casualties $CAS^{(T)}$ using Eq. (29)

next ship count S

display $R^{(\tau)}(S)$ for each ship count S and each transitor τ

display $CAS^{(T)}(S)$ for each ship count S

display $R^{(\tau)}$ for each transitor τ

display $CAS^{(T)}$

end

A.3 DESCRIPTION FOR AREAS

The overall RISKEVAL algorithm for area operations may be summarised as:

begin

collect data

initialise variables

calculate μ using Eq. (39)

for each ship count S from 1 to K_{max}

calculate the threat from 1 mine $T_1(S)$ using Eq. (35)

for each mine remaining i from 0 to n

calculate the *a posteriori* probability for i mines remaining if all mines were initially on ship count S ($P_i(S)$)

calculate the threat from i mines $T_i(S)$ using Eq. (36)

update the risk $R(S)$ from mines initially on ship count S using Eq. (37)

next mine i

next ship count S

for each ship count S from 1 to K_{max}

calculate overall risk R using Eq. (38)
next ship count S

display R (S) for each ship count S
display R

end

A.4 DATA DICTIONARY

This section gives the data dictionary for the symbols (variables) used in this document. Note that NY is the number of computation points across the channel used by NUCEVAL.

Table A-1 - Data Dictionary

Symbol	Definition	Dimension	Type	Units	Limits
T	The total number of transitors through a channel		INTEGER		1 - 99
μ	The average number of times that a patrolling target will encounter a single mine in a time t		REAL		0.0 - 99999.9
σ_{ship}	Standard deviation of navigational error of transiting shipping		REAL	metres	0.0 - 9999.9
C	Channel width		REAL	metres	0.0 - 9999.9
$CAS(T)$	The overall expected casualties after T transits		REAL		0 - 99.9
$CAS^{(T)}(S)$	The expected number of casualties after T transits along the channel caused by mines all initially on ship count S	(1 to K_{max})	REAL		0 - 99.9
F	The total area of a patrol region		REAL	sq. n miles	0.0 - 999999.9
$g(y)$	Probability that a mine located at an across-channel position y encounters a target vessel within the damage width of the mine W_d	(1 to NY)	REAL		0 - 1.0
$h^{\tau}(y)$	For the τ^{th} transit, the probability of a mine at across-channel position y being actuated by the target vessel	(1 to NY)	REAL		0 - 1.0
$h(y)$	The probability of a mine at across-channel position y being actuated by the target vessel	(1 to NY)	REAL		0 - 1.0
i	Counter for number of mines		INTEGER		0 - 999

Symbol	Definition	Dimension	Type	Units	Limits
immobilise	Whether the target vessel is immobilised by a mine strike within the mine damage width W_d		BOOLEAN		TRUE or FALSE
K_{max}	The maximum ship count expected		INTEGER		1 - 99
$L(k)$	The mean probability across the patrol area that a mine is on ship count k	(0 to K_{max}+1)	REAL		0 - 1.0
$L(k,y)$	The probability that a mine at across-channel position y is on ship count k	(0 to K_{max}+1, 1 to NY)	REAL		0 - 1.0
$m'(y)$	The modified pdf that a single mine remains located at an across-channel position y after MCM effort	(1 to NY)	REAL		0 - 1.0
$m(y)$	The *a priori* pdf of a mine being laid at across-channel position y	(1 to NY)	REAL		0 - 1.0
n	The maximum number of mines expected within the channel or area		INTEGER		0 - 999
N_{ships}	The number of patrolling target vessels		INTEGER		1 - 20
NY	The number of computation points across the channel used by NUCEVAL		INTEGER		1 - 999
$PC(S,y)$	Percentage clearance as a fraction for mines initially on ship count S at across-channel position y	(1 to K_{max}, 1 to NY)	REAL		0 - 1.0
$\overline{PC(S)}$	Percentage clearance as a fraction for mines initially on ship count S averaged across the channel / area	(1 to K_{max})	REAL		0 - 1.0
$P_i(S)$	The *a posteriori* probability of i mines initially on ship count S remaining in channel / area	(1 to K_{max})	REAL		0 - 1.0
R	The expected risk to the first target vessel		REAL		0 - 1.0
$R^{(\tau)}$	The expected risk to the τ^{th} transit	(1 to T)	REAL		0 - 1.0
$R^{(\tau)}(S)$	The expected risk to the τ^{th} transit from mines initially on ship count S	(1 to K_{max}, 1 to T)	REAL		0 - 1.0
$R(S)$	The expected risk to the first target vessel from mines initially on ship count S	(1 to K_{max})	REAL		0 - 1.0
$s(u)$	Shipping pdf - probability of transiting shipping being at across-channel position u		REAL		0 - 1.0
$SCDIST(S)$	The *a priori* ship count distribution vector	(1 to K_{max})	REAL		0 - 1.0
t	The time spent patrolling within the area		REAL	hours	0 - 999.9

Symbol	Definition	Dimension	Type	Units	Limits
T_i	Initial threat posed to first transit by i poised mines		REAL		0 - 1.0
$T_i(S)$	The initial threat posed to the first transit by i mines initially on ship count S	(1 to K_{max})	REAL		0 - 1.0
$T_i^{(\tau)}(S)$	The threat posed to the τ^{th} transit by i mines initially on ship count S	(1 to K_{max}, 1 to T)	REAL		0 - 1.0
v	The speed of a patrolling target vessel		REAL	knots	0 - 99.9
W_a	The aggregate actuation width of the target vessel		REAL	metres	0.0 - 9999.9
W_d	Integral of the mine actuation probability over the channel width		REAL	metres	0.0 - 9999.9
y	Across-channel distance; 0 denotes the channel centreline		REAL	metres	-9999.9 - +9999.9

A.5 INPUT PARAMETERS

The input parameters required by the risk evaluation algorithms are given in Table A-2.

Table A-2 - Input parameters

Symbol	Definition	Source	Resolution (Decimal Places)
T	The total number of transitors through a channel	User input	0
σ_{ship}	Standard deviation of navigational error of transiting shipping	User input	0
C	Channel width; note that the channel width should be at least 6 times the σ_{ship}	User Input or from NUCEVAL module	0
F	The total area of a patrol region	User input	0
immobilise	Whether the target vessel is immobilised by a mine strike within the mine damage width W_d	User input	YES or NO
K_{max}	The maximum ship count expected	User input	0
n	The maximum number of mines expected within the channel or area	User input or calculated	0
N_{ships}	The number of patrolling target vessels	User input	0
NY	The number of computation points across the channel used by NUCEVAL	NUCEVAL module	-
$PC(S,y)$	Percentage clearance as a fraction for mines initially on ship count S at across-channel position y	NUCEVAL module	-

Symbol	Definition	Source	Resolution (Decimal Places)
$\overline{PC(S)}$	Percentage clearance as a fraction for mines initially on ship count S averaged across the channel / area	NUCEVAL module	-
$s(u)$	Shipping pdf - probability of transiting shipping being at across-channel position u	User selectable	-
$SCDIST(S)$	The *a priori* ship count distribution vector	User input	3
t	The time spent patrolling within the area	User input	0
v	The speed of a patrolling target vessel	User input	0
W_a	The aggregate actuation width of the target vessel	User input	0
W_d	Integral of the mine actuation probability over the channel width	User input	0

A.6 OUTPUT

The output parameters of the risk evaluation algorithms are detailed in Table A-3.

Table A-3 - Output parameters

Symbol	Definition	Resolution (Decimal Places)
$CAS(T)$	The overall expected casualties after T transits	2
$CAS^{(T)}(S)$	The expected number of casualties after T transits along the channel caused by mines all initially on ship count S	2
R	The expected risk to the first target vessel	3
$R^{(\tau)}$	The expected risk to the τ^{th} transit	3
$R^{(\tau)}(S)$	The expected risk to the τ^{th} transit from mines initially on ship count S	3
$R(S)$	The expected risk to the first target vessel from mines initially on ship count S	3

Annex B

Microsoft Visual Basic source code listing

This Annex lists the Microsoft Visual Basic source code developed by SACLANTCEN to test the developed mine threat algorithms.

```
Option Explicit

' This constant defines the number of calculation points across the channel
Const NY = 201
' The clearance array is used to store the clearance probability across the channel
' as calculated by the NUCEVAL module
Dim PC() As Single
' The ship count vector
Dim L() As Single
' The a priori ship count distribution
Dim SCDIST() As Single
' Whether the target is immobilised on mine strike
Dim immobilise As Boolean
' The maximum ship count to consider
Dim Kmax As Integer
' The calculated mine probability across the channel
Dim m() As Single
' The shipping damage probability density function
Dim g() As Single
' The shipping actuation probability density function
Dim h() As Single
' The entered channel width
Dim C As Single
' The entered mine damage width
Dim Wd As Single
' The entered mine actuation width
Dim Wa As Single
'The maximum number of mines to be considered
Dim n As Integer
' The type of shipping distribution - takes the values NORMAL, UNIFORM or CENTRELINE
```

```
Dim shipping_distn As String
' Switches for program control based on the shipping distribution..
' The normal switch is set to true if the shipping distribution is NORMAL
Dim normal As Boolean
' and the centreline switch is set to true if the shipping distribution is centreline
Dim centreline As Boolean
' Standard deviation of shipping
Dim Sigma_ship As Single
' The calculated threat from a single mine for the current transitor
Dim single_threat As Single
' The number of transitors to consider
Dim Total_tau As Integer
' The expected risk from mines initially on ship count S
Dim RS() As Single
' The expected risk to each transitor
Dim R() As Single
' The expected number of casualties
Dim CAS_S() As Single
' Overall casualties
Dim CAS As Single
' Probability of i mines remaining  (the mine a posteriori distribution)
Dim Pi() As Single
' The counter storing the current ship count being considered
Dim S As Integer

' The resolution across the channel DY = C / (NY - 1)
Dim DY As Single
' The index of the cenre of the channel centre = int(NY / 2) + 1
Dim centre As Single

Sub threat()
' This is the executive to calculate the threat to a transitor
' This is the counter for each transitor
Dim tau As Integer
' This is the counter for the number of mines remaining
Dim i As Integer
' This is the variable for the threat from i mines all initially on ship count S
Dim TiS As Single
' Read in the variables
```

```
get_data
' Calculate the resolution
Resolution
' Dimension all the arrays
set_size
' Read in the clearance and ship count probability data
get_PC
' Get the a priori ship count distribution
get_SCDIST
' Find the damage probability function at all points across the channel...
calculate_g
' Find the actuation probability function at all points across the channel...
calculate_h
' Now loop around all the possible values of S ...
For S = 1 To Kmax
      ' Calculate the mine probability distribution...
      calculate_m
      ' Calculate the ship count vector L for the current ship count...
      initial_L
      ' Loop for number of mines remaining
      For i = 0 To n
            ' Calculate the number of mines remaining
            Call calculate_Pi(i)
            ' Loop for each transitor....
            For tau = 1 To Total_tau
                  ' Find the threat from one mine ...
                  calculate_single_T
                  ' Calculate the threat from i independent mines
                  TiS = 1 - (1 - single_threat) ^ i
                  ' Update the risk
                  RS(S, tau) = RS(S, tau) + Pi(i) * TiS
                  ' Apply the effect of the transit to the ship count distribution....
                  Call update_L(i)
                   ' Next transitor ....
            Next tau
            ' Next mine number
      Next i
      ' Calculate the expected casualties for mines on ship count S
      For tau = 1 To Total_tau
```

```
            CAS_S(S) = CAS_S(S) + RS(S, tau)
        Next tau
        ' Next ship count
Next S
' Average over the ship counts
k_weight
' Write to the screen
output
End Sub

Sub get_data()
' Procedure to read current run data - note that the read statements are used to
' indicate that the data are to be read from the user interface
' The total number of mines remaining to be considered
read n
' The channel width
read C
' The type of shipping distribution
read shipping_distn
' Set the logical variable controlling the type of shipping distribution used
If shipping_distn = "NORMAL" Then
        normal = True
        centreline = False
        ' The shipping standard deviation of navigation error
        read Sigma_ship
ElseIf shipping_distn = "CENTRELINE" Then
        centreline = True
        normal = False
Else
        normal = False
        centreline = False
End If
' The maximum ship count to be considered
read Kmax
' The mine damage width
read Wd
' The target-mine actuation width
read Wa
' The total number of transits
```

```
read Total_tau
' Is the target vessel immobilised?
read immobilise
End Sub

Sub Resolution()
' Subroutine to find the resolution
' Calculate the across-channel distance between calculation points
DY = C / (NY - 1)
' Find the index of the centre of the channel
' (taken as the across-channel origin y = 0)
centre = Int(NY / 2) + 1
End Sub

Sub set_size()
' This subroutine dimensions the arrays
Dim k As Integer
Dim tau As Integer
' Re-dimension the clearance array
ReDim PC(1 To Kmax, 1 To NY)
' Dynamically dimension the a priori ship count probability arrays
ReDim SCDIST(1 To Kmax)
' Dynamically dimension the ship count probability array
ReDim L(0 To Kmax + 1, 1 To NY)
' The mine distribution
ReDim m(NY)
' The ship damage and actuation functions...
ReDim g(NY)
ReDim h(NY)
' The mine probabilities
ReDim Pi(0 To n)
' Dynamically dimension the arrays containing the threat per transitor
ReDim CAS_S(1 To Kmax)
ReDim RS(1 To Kmax, 1 To Total_tau)
ReDim R(1 To Total_tau)
' and initialise these results arrays
For k = 1 To Kmax
      CAS_S(k) = 0
      For tau = 1 To Total_tau
```

```
            RS(k, tau) = 0
            R(tau) = 0
      Next tau
Next k
End Sub

Sub get_PC()
' Procedure to read the clearance data - passed from the NUCEVAL module
Dim iy As Integer
Dim k As Integer
' Read clearance data
For iy = 1 To NY
      For k = 1 To Kmax
            read PC(k, iy)
      Next k
Next iy
End Sub

Sub get_SCDIST()
' This subroutine reads the a priori ship count distribution
Dim k As Integer
Dim sum As Single
' Read the data for each shi count
For k = 1 To Kmax
      read SCDIST(k)
Next k
' Normalise.....
sum = 0
For k = 1 To Kmax
      sum = sum + SCDIST(k)
Next k
For k = 1 To Kmax
      SCDIST(k) = SCDIST(k) / sum
Next k
End Sub

Sub calculate_m()
' This subroutine calculate the mine probability density function for the
' current ship count
```

```
' specified by the variable S
Dim area As Double
Dim iy As Integer
' Initialise the area
area = 0
' Find the area under the (1-clearance) curve using the trapezium rule ....
For iy = 2 To (NY - 1)
      area = area + (1 - PC(S, iy))
Next iy
area = (1 - PC(S, 1)) + (1 - PC(S, NY)) + 2 * area
area = DY * area / 2
' Find the minelay probability distribution
For iy = 1 To NY
      m(iy) = (1 - PC(S, iy)) / area
Next iy
End Sub

Sub calculate_g()
' This routine finds the damage integral
Dim iy As Integer
For iy = 1 To NY
      g(iy) = encounter_prob(Wd / 2, iy)
Next iy
End Sub

Sub calculate_h()
' This routine finds the actuation integral
Dim iy As Integer
For iy = 1 To NY
      h(iy) = encounter_prob(Wa / 2, iy)
Next iy
End Sub

Function encounter_prob(half_width As Single, iy As Integer) As Single
' This function finds the probability of a mine at position iy encountering
' a target within radius half_width
' The value of the integral is dependent on the shipping distribution ....
Dim port_edge As Integer
Dim stbd_edge As Integer
```

```
If centreline Then
      ' The target is always on the channel centreline ...
      If (2 * half_width) > C Then
            encounter_prob = 1
      Else
            ' If the mine lies within the half width of the centre then
            ' attempt to actuate
            If iy >= (centre - Int(half_width / DY)) And
               iy <= (centre + Int(half_width / DY)) Then
                  encounter_prob = 1
            Else
                  encounter_prob = 0
            End If
      End If
ElseIf normal Then
      ' the target distribution is described by a normal distribution ....
      encounter_prob = CumNormDist(((iy - centre) * DY + half_width), 0, Sigma_ship) - _
                       CumNormDist(((iy - centre) * DY - half_width), 0, Sigma_ship)
Else
      ' the target pdf is described by a uniform distribution ....
      ' Then encounter probability is dependent on position in channel.
      port_edge = 1 + Int(half_width / DY)
      stbd_edge = NY - Int(half_width / DY)
      If iy <= port_edge Then
            encounter_prob = ((iy - centre) * DY + half_width + C / 2) / C
            If encounter_prob > 1 Then
                  encounter_prob = 1
            End If
      ElseIf iy >= stbd_edge Then
            encounter_prob = (-(iy - centre) * DY + half_width + C / 2) / C
            If encounter_prob > 1 Then
                  encounter_prob = 1
            End If
      Else
            encounter_prob = 2 * half_width / C
            If encounter_prob > 1 Then
                  encounter_prob = 1
            End If
      End If
```

```
End If
End Function

Sub calculate_single_T()
' This procedure performs the final integration to calculate the threat posed
' by a single mine
Dim iy As Integer
' Initialise the threat
single_threat = 0
' Integrate using the trapezium rule ....
For iy = 2 To (NY - 1)
      single_threat = single_threat + m(iy) * L(1, iy) * g(iy)
Next iy
single_threat = m(1) * L(1, 1) * g(1) + _
                m(NY) * L(1, NY) * g(NY) + _
                2 * single_threat
single_threat = DY * single_threat / 2
End Sub

Sub initial_L()
' This subroutine calculates the ship count vector L after MCM operations for the
' current ship count as specified by the variable S
Dim iy As Integer
Dim k As Integer
' Reset the ship count array
For iy = 1 To NY
      For k = 0 To Kmax + 1
            L(k, iy) = 0
      Next k
Next iy
' Set up the initial L from the clearance achieved by MCM
For iy = 1 To NY
      ' Loop through the ship counts to be considered
      For k = 1 To S - 1
            ' The probability of obtaining exactly (S-k) actuations is equal to the
            ' clearance for ship count S - k minus the clearance for ship count
            ' S-(k-1). Notice that we divide by the probability of clearing the mines
            ' to get the PROPORTION of mines remaining on the ship count k.
            L(k, iy) = (PC(S - k, iy) - PC(S - k + 1, iy)) / (1 - PC(S, iy))
```

```
      Next k
      ' The proportion of mines on ship count S..
      ' Trap overflow errors ...
      If PC(S, iy) > 0.999999 Then
            L(S, iy) = 0
      Else
            L(S, iy) = (1 - PC(1, iy)) / (1 - PC(S, iy))
      End If
Next iy
End Sub

Sub update_L(i As Integer)
' This routine updates the ship count distribution after the transit of a target
Dim iy As Integer
Dim k As Integer
Dim how_far As Single
Dim p_act As Single
' Calculate how far the ship gets through the minefield ....
If immobilise Then
      ' Use formula for how far the target transits through before immobilisation
      how_far = (1 - (1 - single_threat) ^ i) / (i * single_threat)
Else
      ' apply the probability all through the channel
      how_far = 1
End If
For iy = 1 To NY
      ' Modify the value of actuation probability by the probability that the target
      ' has reached the mine
      p_act = how_far * h(iy)
      ' Now update the ship count vector for the transit of the vessel
      L(0, iy) = L(0, iy) + L(1, iy) * p_act
      For k = 1 To S
            L(k, iy) = L(k, iy) * (1 - p_act) + L(k + 1, iy) * p_act
      Next k
Next iy
End Sub

Sub k_weight()
' This subroutine weights the threats for each individual ship count by the
```

```
' a priori ship count distribution
' Counter for the transit number
Dim tau As Integer
' Initialise the overall casualties variable
CAS = 0
' Loop through all the transitors...
For tau = 1 To Total_tau
      ' Initialise the overall threat to this transit
      R(tau) = 0
      ' Loop through all the possible ship counts
      For S = 1 To Kmax
      ' The overall threat to the transit is threat from ship count k
      ' weighted by the a priori ship count distribution
      R(tau) = R(tau) + SCDIST(S) * RS(S, tau)
      Next S
      ' Update the overall number of casualties
      CAS = CAS + R(tau)
Next tau
End Sub

Sub output()
' This routine displays the data - note that the output statements are used to
' indicate that the data are to be displayed on the user interface
Dim tau As Integer
' Write data...
For tau = 1 To Total_tau
      output R(tau)
      For S = 1 To Kmax
            output RS(S, tau)
      Next S
Next tau
output = CAS
For S = 1 To Kmax
      output CAS_S(S)
Next S
End Sub

Sub calculate_Pi(i As Integer)
' This subroutine is a stub where the calculation of the a posteriori mine
```

```
' distribution should be done.  Passed parameter i is the number of mines
' being considered.
Pi(i) = 1
End Sub
```

Document Data Sheet

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Title

Evaluation of mine threat

Abstract

This report describes methods for incorporating ship count distributions into the calculation of both the mine threat to target vessels and the number of expected casualties, prior to and after the execution of mine countermeasures (MCM) operations. The theory is developed for the scenarios of a target transiting along and constrained to a channel, and a target patrolling within an area. Detailed mathematical descriptions are provided of the methods developed to evaluate the mine threat. A comparison is made between the results predicted by the proposed algorithms and those produced by a Monte Carlo simulation. Algorithmic descriptions are given suitable for computer implementation.

Keywords

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